A two-level optical FTTH network design problem

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1 Introduction

In this work, we consider a Fiber To The Home (FTTH) network design problem in a local area with several thousands of subscribers. We present a compact Integer Programming (IP) formulation to employ Column Generation (CG). The columns to be generated are well-defined parts of the FTTH network. The complexity of the pricing problems are established. This on-going work will be completed with a final the implementation.

As mentioned in [1], France Telecom-Orange has decided to use exclusively the existing network infrastructure that contains understreet pipelines. In a local area, thick optical fiber cables, installed into the big-diameter pipelines, connect the *distribution points* to the Optical Line Termination (OLT). At the distibution points, thick fiber cables are split into thinner ones for the final connection to network nodes. At a network node, there is a demand that is the number of subscribers assigned to that node. The connection using thick fiber cables between the OLT and the distribution points is called the *transport network*. The *distribution network* connects a number of network nodes to one distribution point. Every distribution point can serve a limited number of subscribers. A *cluster* is a set of network nodes whose subscribers are served by a certain distribution network. A distribution point may not be in the cluster of its own distribution network. Furthermore, a pipeline may be used simultaneously both in the transport and in a distribution network, but not in more than one distribution network. In this work we consider three main costs; the fixed cost of a distribution point, the costs of the transport network and the distribution networks.

2 Problem description and basic notation

We are given an undirected graph $G = (V, E)$, the set V of network nodes $0, \dots, n$ with demands $\rho_1, \dots, \rho_n \in \mathbb{N}_0$, and the set E of edges $1, \dots, m$ with lentghs $l_1, \dots, l_m \in \mathbb{Q}^+$, and with transport line qualifications $t_1, \dots, t_m \in \{0, 1\}$. Node 0 is the OLT.

In the FTTH network design, there are four hierarchical decisions. First, a transport line, denoted by η, is determined by choosing a subset of edge in the set $\{e \in E : t_e = 1\}$.

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Secondly, given η, the set of distribution points, denoted by D_η, is selected among the nodes in the subgraph $G[\eta]$. Thirdly, for every distribution point $i \in D_\eta$, a distribution line, denoted by $\tau_i \subset E$, is selected. Lastly, the cluster of τ_i, denoted by Γ_{τ_i}, is selected from the nodes in subgraph $G[\tau_i]$ in a way that every node in $G[\tau_i]$ except node i, is in Γ_{τ_i}, but node i may not be. In a feasible FTTH network, the OLT is in $G[\eta]$, $G[\eta]$ is connected, $G[\tau_i]$ is connected for every $i \in D_\eta$, and τ_i must have at least one edge incident to node i.

Operational requirements force that every node with positive demand must be in exactly one cluster, i.e. $|\{i \in D\eta : j \in \Gamma_{\tau_i}\}| = 1$ for all $j \in V$ with $\rho_j > 0$. Moreover, every pipeline is used in at most one distribution line, i.e. $|\{i \in D_\eta : e \in \tau_i\}| \leq 1$ for all $e \in E$. Technological constraints require that $\sum_{j \in \Gamma_{\tau_i}} \rho_j \leq \mathcal{C}$ for every distribution line τ_i. The fixed cost of a distribution point is C_p, and the unit lengths of the transport line and of the distribution line cost C_t and C_d.

3 Mathematical formulation and complexity results

We present a compact IP formulation in this section. The binary variable x_η for every $\eta \in \mathcal{T}$ indicates that transport line η is selected. Assigning node i as a distribution point is indicated by variables y_i, for all $i \in V$. Finally, the binary variable $\kappa_{i,\tau}$ is equal to 1 if the distribution line $\tau \in \mathcal{D}_i$ is chosen for $i \in V$. The $0-1$ inclusion parameters $\theta_{i,\eta}$, $\gamma_{e,\tau}$, and $\delta_{j,\tau}$ are 1 if node i is in the subgraph $G[\eta]$, $e \in \tau$, and $j \in \Gamma_\tau$ respectively. The costs of transport line η and ditribution line τ are denoted by $c_\eta = C_t \sum_{e \in \eta} l_e$ and $c_\tau = C_d \sum_{e \in \tau} l_e$.

$$(P) \qquad \text{Min} \qquad C_p \sum_{i \in V} y_i + \sum_{\eta \in \mathcal{T}} c_\eta x_\eta + \sum_{i \in V} \sum_{\tau \in \mathcal{D}_i} c_\tau \kappa_{i,\tau} \tag{1}$$

subject to:

$$\sum_{\eta \in \mathcal{T}} x_\eta = 1, \tag{2}$$

$$\sum_{\eta \in \mathcal{T}} \theta^i_\eta x_\eta \geq y_i \ , \qquad \forall i \in V \tag{3}$$

$$\sum_{\tau \in \mathcal{D}_i} \kappa_{i,\tau} = y_i, \qquad \forall i \in V \tag{4}$$

$$\text{sgn}(\rho_j) \leq \sum_{i \in V} \sum_{\tau \in \mathcal{D}_i} \delta_{j,\tau} \kappa_{i,\tau} \leq 1, \qquad \forall j \in V \tag{5}$$

$$\sum_{i \in V} \sum_{\tau \in \mathcal{D}_i} \gamma_{e,\tau} \kappa_{i,\tau} \leq 1, \qquad \forall e \in E \tag{6}$$

$$y_i, x_\eta, \kappa_{i,\tau} \in \{0,1\}, \quad \forall i \in V, \ \forall \tau \in \mathcal{D}_i \ \forall \eta \in \mathcal{T} \tag{7}$$

The model (P) selects exactly one transport line (constraints (2)) with proper distribution points (constraints (3)) such that every distribution point has its distribution line (constraints (4)). Operational requirements are expressed in the constraints (5)-(6).

In the CG procedure, given an optimal solution (y^*, x^*, κ^*) to the restricted master problem, dual variables (ω^*, ϕ^*, σ^*, π^*, λ^*) are associated with the constraints (2)-(6) in the respected order. Then the pricing problems for transport lines and for distribution lines are defined below.

PROBLEM: TRANSPORT LINE PRICING PROBLEM (*TLPP*)

INSTANCE: Given an undirected graph $G = (V, E)$. Every node $i \in V$ has a *non-negative* prize ϕ_i^* and every edge $e \in E$ has a *non-negative* cost $C_t l_e$. A transport line η is a set of edges inducing a connected subgraph $G[\eta]$ that includes node 0.

Objective: Find a transport line η *minimizing* the cost $\sum_{e \in \eta} C_t l_e - \sum_{i \in V(G[\eta])} \phi_i^*$.

Theorem 1. *It is NP-hard to approximate the TLPP within any constant factor.*

Proof. Given the same data of the *TLPP*, The Price Collecting Steiner Tree Problem with Net Worth maximization amounts to find a rooted tree in G by maximizing the profit. We conclude that both problems are equivalent, since the profit is the negative of the cost of a transport line (See [2]) and an optimal transport line is a tree rooted at the OLT. In [2], it is mentioned that the same statement in the theorem for the aforementioned problem is shown to be true in [3]. Then the theorem follows the equivalence of two problems. □

PROBLEM: DISTRIBUTION LINE PRICING PROBLEM OF NODE i ($DLPP_i$)

INSTANCE: Given an undirected graph $G = (V, E)$ with a distinguished node i, prizes π_j^* for every $j \in V$, and *non-negative* costs $C_d l_e - \lambda_e^*$ for every $e \in E$. A distribution line $\tau \subseteq E$ with the cluster Γ_τ satisfies $i \in V(G[\tau])$ and $\sum_{j \in \Gamma_\tau} \rho_j \leq \mathcal{C}$. It is not necessary that $i \in \Gamma_\tau$.

Objective: Find a distribution line τ *minimizing* $\sum_{e \in \tau}(C_d l_e - \lambda_e^*) - \sum_{j \in \Gamma_\tau} \pi_j^*$.

Proposition 2. *An algorithm solving TLPP solves $DLPP_i$ instances with $\mathcal{C} \geq \sum_{j \in V} \rho_j$.*

Proof. Firstly, given such a $DLPP_i$ instance, let us obtain *TLPP* instances by making all weights non-negative. If $K = \min_{j \in V}\{\pi_j^*\} < 0$, all weights are decreased by K. Second, in the $DLPP_i$ we may have no *root node* if node i is not in the cluster, but we need a root node in the *TLPP* instances. So first we let node i to be the root node, and solve the corresponding *TLPP* instance. To check if we can have a distribution line with smaller objective value without node i in the cluster, we obtain several *TLPP* instances, each having a root node that is one unselected neighbor of node i in the optimal solution. In these instances all edges incident to node i are removed, since we want to see the solution excluding node i. Once we have the optimal solutions of the derived $TLPP$ instances, some objective values should be corrected. We add K to the objective value of the instance in which i is the root node and weights are modified. We add the cost of the edge between node i and its neighbor, if that neighbor is the root node. Among all optimal solutions of the derived *TLPP* instances, the one with the smallest corrected objective value is the optimal solution of the $DLPP_i$ instance. □

References

[1] M. CHARDY, M.C. COSTA, A. FAYE, M. TRAMPONT (2012). *Optimizing splitter and fiber location in a multilevel optical FTTH network.* Eur J Oper Res 222, pp. 430–440.

[2] D.S. JOHNSON, M. MINKOFF, S. PHILIPS (2000). *The prize collecting Steiner tree problem: Theory and practice.* Proceedings of the 11th ACM-SIAM Symposium on Discrete Mathematics, San Fransisco, CA, 2000, pp. 760–769.

[3] J. FEIGENBAUM, C. PAPADIMITRIOU, S. SHENKER (2001). *Sharing the cost of multicast transmissions.* J Comp and Sys Scien 63, pp. 21–41.